1. If \( y = x \sin 2x \), prove that \( x \frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + \frac{2y}{x} + 4xy = 0 \)

2. Given that \( y = e^x - e^{-x} \), show that \( \left( \frac{dy}{dx} \right)^2 - y^2 - 4 = 0 \)

3. Given that \( v = \sqrt{\sin u} \), show that \( 4v^3 \frac{d^2v}{du^2} + v^4 + 1 = 0 \)

4. Given \( y = e^{-x} \cos x \), show that \( \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0 \).

5. Given that \( y = \frac{\sin kx}{1 + \cos kx} \), where \( k \) is a positive integer, show that \( \sin kx \frac{d^2y}{dx^2} = k^2 y^2 \).

6. Given \( y(2 - x) = 3 \), show that \( 3 \frac{d^2y}{dx^2} - 2y \frac{dy}{dx} = 0 \).

7. Given \( y = (1 + 4x)e^{-2x} \), prove that \( \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 4y = 0 \).

8. Let \( y = \sqrt{\cos x} \), show that \( 4y^3 \frac{d^2y}{dx^2} + y^4 + 1 = 0 \).

9. Given \( (1 + x^2)y^2 = 1 - x^2 \), show that \( \left( \frac{dy}{dx} \right)^2 = \frac{1 - y^4}{1 - x^2} \).

10. Form a differential equation from \( y = Ax^3 + \frac{B}{x^2} - 6x \), \( x > 0 \).

11. Form a differential equation from \( y = Ax^3 + \frac{B}{x^2} - 6 \), \( x > 0 \).

12. \( y \sin^{-1}3x = \sqrt{1 - 9x^2} \), show that \( (1 - 9x^2) \frac{dy}{dx} + 3y^2 + 9xy = 0 \)

13. Given that \( y = x^n[A \cos(\ln x) + B \sin(\ln x)] \), where \( A \) and \( B \) are constants, show that

\[ x^2 \frac{d^2y}{dx^2} + (1 - 2n)x \frac{dy}{dx} + (1 + n^2)y = 0 \]

14. Given that \( y = \sin^{-1}x \), show that \( (1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0 \)

15. Let \( y = 5e^{(\sqrt{3}-2)x} + 3e^{-(\sqrt{3}+2)x} \), show that \( \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + y = 0 \).