Area

**Question 1**

A trapezium ABCD with AB//DC is divided into four triangles by its diagonals.

Let the triangles adjacent to the parallel sides have areas $a$ and $b$.

Find the area of the trapezoid in terms of $a$ and $b$.

**Question 2**

ABCD is a rectangle.
The areas of the right angled triangles are $a$, $b$, $c$, as in the figure.

Find the area of the triangle, $S$, in terms of $a$, $b$, $c$.

**Question 3**

ABC is a triangle.
BD and CE cut at F.
If area of $\triangle BEF = a$, area of $\triangle BFC = b$, area of $\triangle CFD = c$, find the area of the quadrilateral AEFD.
Answers

Question 1

Let the two diagonals AC and BD meets at E.

Let \( DE = x, \ BE = y \)

\[ \frac{c}{a} = \frac{x}{y}, \quad \frac{b}{d} = \frac{x}{y} \]

:. \( \frac{c}{a} = \frac{b}{d} \) \hspace{1cm} (1)

Area of \( \triangle ACD = \) Area of \( \triangle BCD \)

\:. \( c + b = d + b \)

:. \( c = d \) \hspace{1cm} (2)

(2) ↓ (1), \( c = d = \sqrt{ab} \)

:. Area of trapezium \( ABCD = a + b + c + d = a + b + 2\sqrt{ab} = (\sqrt{a} + \sqrt{b})^2 \)

Question 2

Let

\( p = DC, \ q = AD \)

\( x = FB, \ y = BE \)

\[ x = p = \frac{2a}{q}, \quad y = q = \frac{2c}{p} \]

\[ b = \frac{1}{2}xy = \frac{1}{2}\left( p = \frac{2a}{q} \right) \left( q = \frac{2c}{p} \right) \]

\[ 2b = pq - 2a - 2c - \frac{4ac}{pq} \]

\[ pq - 2(a + b + c) - \frac{4ac}{pq} = 0 \]

\[ (pq)^2 - 2(a + b + c)(pq) - 4ac = 0 \]

Consider only the positive root, we have:

\[ pq = \frac{2(a + b + c) + \sqrt{[2(a + b + c)]^2 - 4(1)(-4ac)}}{2(1)} = (a + b + c) + \sqrt{(a + b + c)^2 + 4ac} \]

\:. \( S = pq - (a + b + c) = \sqrt{(a + b + c)^2 + 4ac} \)
Question 3

\[
a + x = \frac{BF}{y} = \frac{b}{c} \\
by = ca + cx \quad (1)
\]

\[
c + y = \frac{CF}{x} = \frac{b}{a} \\
bx = ac + ay \quad (2)
\]

Solve (1), (2), we have:

\[
x = \frac{ac(a + b)}{b^2 - ac}, \quad y = \frac{ac(b + c)}{b^2 - ac}
\]

\[
\therefore \text{Area of quad. } AEFD = x + y = \frac{ac(a + 2b + c)}{b^2 - ac}
\]