QUEEN'S COLLEGE
Mock Examination 2006-2007
Mathematics & Statistics

Secondary 7A & 7S

Date: 27th February, 2007.
Time: 8:30 am – 11:30 am

Instructions:

1. This paper consists of Section A and Section B.
2. All answers should be written in the answer book provided.
3. Answer any FOUR questions in Section B.
4. Unless otherwise specified, numerical answers should either be exact or correct to 4 decimal places.

DO NOT turn over this question paper until you are told to do so.
Section A (40%)  
Answer ALL questions in this section.  
1. Let \( y = 3e^{2x} - 13e^x - 5x + 2 \).  
   a) Find \( \frac{dy}{dx} \). \( \quad \) (2 marks)  
   b) Find the value of \( x \) at which \( \frac{dy}{dx} = 0 \). \( \quad \) (4 marks)  
2. In how many ways can 4 men be chosen from 5 Americans and 6 Chinese so as to include:  
   a) only one Chinese? \( \quad \) (2 marks)  
   b) at least one Chinese? \( \quad \) (4 marks)  
3. Expand the first four terms of \( \left( 3 + \frac{1}{x^2} \right)^4 \) in descending powers of \( x \).  
   a) State the range of values of \( x \) for which the above expansion is valid. \( \quad \) (6 marks)  
   b) Find the general term of this expansion and hence find the \( x^{-12} \) coefficient. \( \quad \) (6 marks)  
4. A noise detector was used to detect the noise level (in decibels) during a concert in the Hong Kong Stadium. The results were recorded as follows:  
   \[
   \begin{array}{cccccccc}
   82 & 74 & 88 & 66 & 58 & 74 & 78 & 84 & 96 & 76 \\
   62 & 68 & 72 & 92 & 86 & 76 & 52 & 76 & 82 & 78 \\
   \end{array}
   \]  
   a) Copy and complete the following stem and leaf diagram for the above data:  
   Stem (in 10 decibels) \( \quad \) Leaf (in 1 decibel) \( \quad \)
   \[
   \begin{array}{cccccccc}
   5 & & & & & & & \\
   6 & & & & & & & \\
   7 & & & & & & & \\
   8 & & & & & & & \\
   9 & & & & & & & \\
   \end{array}
   \]  
   b) Find the median and the interquartile range of the above data. \( \quad \) (5 marks)
5. Two office assistants, John and David, are responsible for dispatching documents. Between them, John processes 45% of the documents and David processes 55%. The probabilities that a document is placed in a wrong tray by John and David are 0.05 and 0.06 respectively.

a) A document selected at random from a tray is found to be in a wrong tray. What is the probability that it was processed by John?
   (3 marks)

b) Two documents selected at random from a tray are found to be in a wrong tray. What is the probability that both documents were processed by the same office assistant?
   (3 marks)

6. The velocity of a moving particle at time \( t \) is given by \( v = \frac{t}{\sqrt{t^2 + 2}} \), where \( t \) is in seconds. Suppose, at \( t = 2 \), the particle is 3 units from its initial position. Find the distance travelled by the particle in 5 seconds. (Leave your answer in surd form)
   (5 marks)

7. A survey was conducted to collect data on the speeds of cars at a road junction. The data showed that 10.2% of the cars travelled at over 90 km/hr and 2.5% of the cars travelled at below 60 km/hr. It is believed that the speeds of the cars are normally distributed.

a) Find the mean and standard deviation of the speeds of the cars.
   (6 marks)

b) What is the percentage of cars traveling at more than 70 km/hr?
   (Give the answer correct to the nearest %)
   (6 marks)

End Of Section A
Section B (60%)
Answer any FOUR questions from this section. Each question carries 15 marks.

8. Let \( f(x) = \ln x \), where \( 1 \leq x \leq 2 \) and \( I = \int_1^2 f(x) \, dx \).

a) (i) Find the estimate \( I_5 \) of \( I \) by using the trapezoidal rule with 5 sub-intervals.

(ii) State whether \( I_5 \) in (a)(i) is an over-estimate or under-estimate of \( I \). Explain your answer briefly. (6 marks)

b) (i) Find \( \frac{d}{dx}(x \ln x) \).

(ii) Hence evaluate \( I \). (4 marks)

c) (i) If \( y = \log_{10} x \), show that \( y = \frac{\ln x}{\ln 10} \).

(ii) Hence evaluate \( \int \log_{10} x \, dx \). (5 marks)

9. Let \( f(x) = e^{2x} - ae^x + b \) and \( g(x) = 3(5^{x+2}) - 3 \) where \( a \) and \( b \) are constants. Page(9) shows a sketch of \( y = g(x) \). It is known that \( y = f(x) \) and \( y = g(x) \) have only 1 point of intersection at the origin. Furthermore, \( y = f(x) \) passes through \( (\ln 3, 2) \).

(a) Find the values \( a \) and \( b \). (2 marks)

(b) (i) Find another \( x \) - intercept of \( f(x) \).

(ii) Find the stationary point(s) of \( y = f(x) \) and determine their nature. (6 marks)

c) Sketch \( y = f(x) \) on page (9). Indicate the asymptote and the points where the curve cuts the axes and \( y = g(x) \). (3 marks)

d) Find the area bounded by the curve \( y = f(x) \), \( y = g(x) \), the line \( x = 0 \) and \( x = 1 \). (4 marks)
10. It is given that A, B and C are three events with \( P(A) = \frac{1}{3} \), \( P(B) = \frac{1}{2} \),
\( P(C) = \frac{2}{5} \), \( P(A \cup B) = \frac{13}{18} \), \( P(A \cup C) = \frac{17}{30} \) and \( P(B \cap C) = \frac{2}{9} \).

(a) Find the following probabilities:
(i) \( P(A \cap B) \),
(ii) \( P(C \mid B) \),
(iii) \( P(A \mid C) \). (7 marks)

(b) Are the three events A, B and C mutually exclusive? (2 marks)

(c) If, in addition, \( P(A \cap B \cap C) = \frac{1}{18} \), find the following probabilities:
(i) \( P(A \cup B \cup C) \),
(ii) \( P(A \cap B \cap C \mid A \cup B \cup C) \). (6 marks)

11. An unknown disease is spread largely by carriers, that is individual who can transmit the disease but exhibit no overt symptoms. One of the local hospitals develops a new medicine to cure the disease. Let \( y(t) \) be the number of carriers (in hundred) and it can be modelled by \( y(t) = ae^{bt} \), where \( t \) is the number of days since the medicine becomes available, \( a \) and \( b \) are constants.

The following table shows the values of \( y(t) \) at different times.

<table>
<thead>
<tr>
<th>( t )</th>
<th>2.25</th>
<th>3.24</th>
<th>12.25</th>
<th>15.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(t) )</td>
<td>37</td>
<td>35</td>
<td>25</td>
<td>23</td>
</tr>
</tbody>
</table>

(a) (i) Express \( y(t) \) as a linear function of \( \sqrt{t} \).
(ii) Use the graph paper on p.10 to estimate graphically the values of \( a \), (correct to nearest integer) and \( b \), (correct to 1 decimal place).
(iii) Use the values of \( a \) and \( b \) obtained in a (ii) and find the value of \( t \), correct to 2 decimal places, when the number of carriers is one fifth of the number at the beginning since the medicine becomes available. (9 marks)

11 (b) Let \( x(t) \) be the total number of people carrying the disease (in hundred). An authority believes that the disease spreads at a rate
(Continued for No.11b)
according to the following function:
\[ \frac{dx(t)}{dt} = \frac{1}{\sqrt{t}} y(t) \]
Use the results in part (a), find the total number of people carrying the disease 4 days after the medicine becomes available. (Correct the answer to 2 decimal places) (6 marks)

12. From a survey, it was found that the times required to travel from Causeway Bay to Central by the Mass Transit Railway (MTR) follow a normal distribution with mean 10 minutes and standard deviation 4 minutes; also, the times required for the same trip by bus follow another normal distribution with mean 15 minutes and standard deviation 5 minutes. Assume that both of the travel times are independent of each other.

(a) Suppose Mr. Chan gets on an MTR or a bus at 8:40 am at Causeway Bay every morning. He has to arrive at Central before 9.00 am. Find the probability that he will be late on a given day if he travels
(i) by MTR,
(ii) by bus.
Which vehicle do you suggest him to take? (6 marks)

(b) Suppose on the average Mr. Chan has a chance of \( \frac{1}{3} \) to travel by bus and \( \frac{2}{3} \) by MTR.
(i) Find the probability that he will be late on a given day.
(ii) Find the probability that he will be late exactly twice on four consecutive days. (4 marks)

(c) The travel times for the same trip by taxi also follow a normal distribution with mean \( \mu \) minutes and standard deviation 3 minutes. It is known that Mr. Chan has a chance of 0.01 to be late if he travels by taxi. (i) Find \( \mu \).
(ii) If he takes taxi for all the 20 working days in this month, find the expected number of days for him to be on time. (5 marks)
13. In a magazine shop, the daily demand of a certain magazine M is believed to follow a Poisson distribution. However, there are two claims about the mean of the distribution, one is 2.5 and the other is 3.2. The shop owner wanted to check which distribution fits the observed data better. He then recorded the number of magazine M sold in 100 days.

The following table summarises his record:

<table>
<thead>
<tr>
<th>Number of magazine M sold</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed frequency (days)</td>
<td>7</td>
<td>18</td>
<td>24</td>
<td>22</td>
<td>14</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

(a) (i) Using the Poisson distributions with means 2 and 3.2 respectively, fill in the missing expected frequencies in Table 1 on P.11 (4 marks)

(ii) State which distribution fits the observed data better. (1 mark)

(b) Suppose the daily sales of a magazine follow a Poisson distribution with mean 3.2.

(i) Every day the shop owner keeps the stock of the magazine to 4 copies. Find the probability that

1. there are not enough copies for sale in a day, (3 marks)
2. there are not enough copies for sale in exactly 3 days out of a week. (3 marks)

(ii) If the shop owner wants to ensure that the probability of not having enough copies for sale is less than 0.05 each day, what is the minimum stock of the magazine that he should keep each day? (4 marks)

END OF PAPER
### Table: Area under the Standard Normal Curve

<table>
<thead>
<tr>
<th>z</th>
<th>0.00</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
<th>0.06</th>
<th>0.07</th>
<th>0.08</th>
<th>0.09</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0000</td>
<td>0.0040</td>
<td>0.0080</td>
<td>0.0120</td>
<td>0.0160</td>
<td>0.0199</td>
<td>0.0239</td>
<td>0.0279</td>
<td>0.0319</td>
<td>0.0359</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0398</td>
<td>0.0438</td>
<td>0.0478</td>
<td>0.0517</td>
<td>0.0557</td>
<td>0.0596</td>
<td>0.0636</td>
<td>0.0675</td>
<td>0.0714</td>
<td>0.0753</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0793</td>
<td>0.0832</td>
<td>0.0871</td>
<td>0.0910</td>
<td>0.0948</td>
<td>0.0987</td>
<td>0.1026</td>
<td>0.1064</td>
<td>0.1103</td>
<td>0.1141</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1170</td>
<td>0.1217</td>
<td>0.1265</td>
<td>0.1312</td>
<td>0.1358</td>
<td>0.1404</td>
<td>0.1450</td>
<td>0.1495</td>
<td>0.1540</td>
<td>0.1585</td>
</tr>
<tr>
<td>0.4</td>
<td>0.1621</td>
<td>0.1668</td>
<td>0.1715</td>
<td>0.1761</td>
<td>0.1806</td>
<td>0.1851</td>
<td>0.1896</td>
<td>0.1941</td>
<td>0.1985</td>
<td>0.2029</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2073</td>
<td>0.2117</td>
<td>0.2160</td>
<td>0.2203</td>
<td>0.2246</td>
<td>0.2288</td>
<td>0.2331</td>
<td>0.2373</td>
<td>0.2415</td>
<td>0.2457</td>
</tr>
<tr>
<td>0.6</td>
<td>0.2499</td>
<td>0.2541</td>
<td>0.2582</td>
<td>0.2623</td>
<td>0.2664</td>
<td>0.2705</td>
<td>0.2746</td>
<td>0.2787</td>
<td>0.2828</td>
<td>0.2869</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2909</td>
<td>0.2950</td>
<td>0.2990</td>
<td>0.3030</td>
<td>0.3070</td>
<td>0.3110</td>
<td>0.3149</td>
<td>0.3188</td>
<td>0.3227</td>
<td>0.3266</td>
</tr>
<tr>
<td>0.8</td>
<td>0.3304</td>
<td>0.3343</td>
<td>0.3381</td>
<td>0.3419</td>
<td>0.3457</td>
<td>0.3494</td>
<td>0.3531</td>
<td>0.3568</td>
<td>0.3604</td>
<td>0.3641</td>
</tr>
<tr>
<td>0.9</td>
<td>0.3678</td>
<td>0.3714</td>
<td>0.3750</td>
<td>0.3786</td>
<td>0.3821</td>
<td>0.3856</td>
<td>0.3891</td>
<td>0.3925</td>
<td>0.3959</td>
<td>0.4003</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4037</td>
<td>0.4070</td>
<td>0.4103</td>
<td>0.4136</td>
<td>0.4169</td>
<td>0.4202</td>
<td>0.4235</td>
<td>0.4267</td>
<td>0.4300</td>
<td>0.4333</td>
</tr>
<tr>
<td>1.1</td>
<td>0.4365</td>
<td>0.4397</td>
<td>0.4429</td>
<td>0.4461</td>
<td>0.4493</td>
<td>0.4524</td>
<td>0.4556</td>
<td>0.4587</td>
<td>0.4619</td>
<td>0.4650</td>
</tr>
<tr>
<td>1.2</td>
<td>0.4681</td>
<td>0.4712</td>
<td>0.4743</td>
<td>0.4774</td>
<td>0.4804</td>
<td>0.4834</td>
<td>0.4864</td>
<td>0.4894</td>
<td>0.4924</td>
<td>0.4953</td>
</tr>
<tr>
<td>1.3</td>
<td>0.4982</td>
<td>0.5012</td>
<td>0.5041</td>
<td>0.5070</td>
<td>0.5099</td>
<td>0.5127</td>
<td>0.5156</td>
<td>0.5184</td>
<td>0.5212</td>
<td>0.5240</td>
</tr>
</tbody>
</table>

**Note:** An entry is the table is the proportion of the area under the entire curve which is between \( z = 0 \) and a positive value of \( z \). Areas for negative values of \( z \) are obtained by symmetry.

**Diagram:**

![Area under the Standard Normal Curve](image)

\[ A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \]
If you attempt Question No.9, fill in your class and class No. as shown above and put this sheet inside your answer book.
If you attempt Question No.11, fill in your class and class No. as shown above and put this sheet inside your answer book.
If you attempt Question No.13, fill in your class and class No. as shown above and put this sheet inside your answer book.

Table 1. Observed and expected frequencies of numbers of magazine M sold

<table>
<thead>
<tr>
<th>Number of Magazine M sold</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Frequency</td>
<td>7</td>
<td>18</td>
<td>24</td>
<td>22</td>
<td>14</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td>Expected Frequency*: Po(2.5)</td>
<td>8.2</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
<tr>
<td>Expected Frequency*: Po(3.2)</td>
<td>4.1</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
<td>( )</td>
</tr>
</tbody>
</table>

* correct to 1 decimal place.
Rough Work