QUEEN’S COLLEGE
Half-yearly Examination 2005-2006
Mathematics & Statistics

Secondary 6

Date: 16/01/2006

Time: 8:30am – 10:30 am

Instructions:

1. Answer ALL questions in Section A and Section B.
2. All answers should be written in the answer book provided.
3. Unless otherwise specified, numerical answers should either be exact or correct to 4 decimal places.

DO NOT turn over this question paper until you are told to do so.
Section A (60 marks)

1. Find the number of integers between 1000 and 4000 which can be formed by using digits 1, 2, 3, 4.
   a) if each digit may be used only once. (8 marks)
   b) if each digit may be used more than once.

2. In the expansion of \((x + a)^4(x - 2a)^5\), the coefficients of \(x^4\) and \(x^7\) are equal and non-zero. Evaluate the value of \(a\). (8 marks)

3. Solve the following equations for \(x\):
   a) \(25^x = 23(5^x) + 50\) (8 marks)
   b) \(\log_x(x + 1) + \log_x(3x - 1) = 5\)

4. a) Assume \(x\) is so small that \(x\) and higher powers of \(x\) can be neglected, expand the following as infinite series as far as the \(x^3\) term:
      i) \(e^x\)
      ii) \(\frac{1 + ax}{1 + bx}\)

   b) Find the values of \(a\) and \(b\) if \(e^x = \frac{1 + ax}{1 + bx}\) (8 marks)

5. Suppose that students are required to study four subjects out of ten. The ten subjects are classified into 2 groups, each containing 5 subjects. Find how many subject combinations are possible:
   a) if a student must take 2 subjects from each group; (8 marks)
   b) if a student must take at least 1 subject from each group.

6. Given: \(f(x) = \sqrt{x}\) and \(g(x) = 5e^x\).
   Determine the composite function \(f[g(x)]\) and \(g[f(x)]\).
   In each case, state the domains and ranges of the composite functions. (10 marks)

7. The value of a car depreciates by 20% per year. The value of a new car is $300000. Find the value of the car after being used for 2 years if the depreciation is compounded
   a) monthly, (10 marks)
   b) daily (assume 365 days in a year),
   c) continuously.
Section B (40 marks)
Answer ALL questions from this section. Each question carries 20 marks.

8 a) Evaluate the following limits:

i) \( \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \) (3 marks)

ii) \( \lim_{x \to 2} \frac{4-x^2}{3-x^2+5} \) (3 marks)

iii) \( \lim_{x \to 4} \frac{1}{h} \) (3 marks)

b) i) Find, by using the first principle, the derivative of the function: \( g(x) = (x+1)^2 \) with respect to \( x \). (4 marks)

ii) Find \( g'(0) \) and \( g'(-1) \). (2 marks)

iii) With the help of a sketch of the graph of \( g(x) = (x+1)^2 \), interpret the result of (ii) geometrically. (5 marks)

9) If you attempt this question, put the graph paper provided inside your answer book.

The population \( P \), in millions, of a small country was recorded in the January of various years and the results are shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>12.3</td>
<td>13.4</td>
<td>15.1</td>
<td>17.1</td>
</tr>
</tbody>
</table>

Given that \( P = 10 + ab^t \) where \( t \) is the time measured in years from January 1980 and \( a \) and \( b \) are constants, express \( \log_a(P-10) \) as a linear function of \( t \). (2 marks)

a) Copy and completing the following table on your answer book.

<table>
<thead>
<tr>
<th>( t )</th>
<th>3</th>
<th>9</th>
<th>15</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>12.3</td>
<td>13.4</td>
<td>15.1</td>
<td>17.1</td>
</tr>
<tr>
<td>( \log_a(P-10) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Give your answers correct to 2 significant figures in the above table. (2 marks)
9b) Draw the graph of \( \log_a (P-10) \) against \( t \) for \( 0 \leq t \leq 25 \). (4 marks)

9c) Use your graph to calculate
   i) the values of \( a \) and \( b \); (5 marks)
   ii) the population which was recorded in the year of 1980. (3 marks)

9d) Find the year in which the population will reach 19.4 million. (4 marks)

*** End ***