1. Write your class, class number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 13 marks.

3. Attempts ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answer in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your class and class number on each sheet and put them inside this book.

4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should either be exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

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<table>
<thead>
<tr>
<th>Question No.</th>
<th>Marks</th>
<th>Section A Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td></td>
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<tr>
<td>2-3</td>
<td>7</td>
<td></td>
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<tr>
<td>4</td>
<td>4</td>
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<tr>
<td>5-6</td>
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<td>7</td>
<td>4</td>
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<td>8</td>
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<td>9</td>
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<td>11</td>
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<table>
<thead>
<tr>
<th>Question No.*</th>
<th>Marks</th>
<th>Section B Total</th>
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</table>

*To be filled in by the candidate.*
### FORMULAS FOR REFERENCE

<table>
<thead>
<tr>
<th>Shape</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPHERE</td>
<td>$4\pi r^2$</td>
<td>$\frac{4}{3}\pi r^3$</td>
</tr>
<tr>
<td>CYLINDER</td>
<td>$2\pi rh$</td>
<td>$\pi r^2h$</td>
</tr>
<tr>
<td>CONE</td>
<td>$\pi rl$</td>
<td>$\frac{1}{3}\pi r^2h$</td>
</tr>
<tr>
<td>PRISM</td>
<td></td>
<td>$\text{base area} \times \text{height}$</td>
</tr>
<tr>
<td>PYRAMID</td>
<td></td>
<td>$\frac{1}{3}\times\text{base area} \times \text{height}$</td>
</tr>
</tbody>
</table>

### Section A(1) (33 marks)

Answer ALL questions in this section and write your answers in the space provided.

1. Simplify $\frac{(2x^3y^{-1})^3}{2y^{-2}}$ and express your answer with positive indices. (3 marks)

   $$\frac{(2x^3y^{-1})^3}{2y^{-2}} = \frac{8x^9y^{-3}}{2y^{-2}}$$
   $$= 4x^9y^{2-3}$$
   $$= 4x^9y^{-1}$$
2. If \( a:b = 3:2 \) and \( b:c = 4:3 \).

(a) Find \( a:b:c \).

(b) Find the value of

\[
\frac{a+2b-3c}{a+c} = \frac{3}{2} \quad \text{(3 marks)}
\]

\[
b : c = \frac{4}{3}
\]

\[
a : b : c = 6 : 4 : 3 \quad 1 \text{A}
\]

Method 1

\[
a : b = \frac{3}{2} \quad \frac{b}{c} = \frac{4}{3}
\]

\[
a = \frac{3}{2} b \quad c = \frac{3}{2} b
\]

\[
a+2b-3c \quad a+c = \frac{\frac{3}{2}b+2b-\frac{9}{2}b}{\frac{3}{2}b+\frac{3}{2}b} = \frac{5}{9}
\]

Method 2

Let \( a = 6k \), \( b = 4k \), \( c = 3k \) \( 1 \text{M} \)

\[
a+2b-3c \quad a+c = \frac{6k+8k-9k}{6k+9k} = \frac{5}{9} \quad 1 \text{A}
\]

3. Johnson's weight is measured to be 60 kg correct to the nearest kg. Jacky's weight is measured to be 64 kg with percentage error of 6%.

(a) Find the maximum absolute error of each of the above measurements.

(b) Is it a must that Jacky is heavier than Johnson? Explain your answer. \( 4 \text{ marks} \)

(a) Max. absolute error of Johnson's weight = 0.5 kg

Max. absolute error of Jacky's weight

\[
= \frac{64}{0.06} = 3.84 \text{ kg}
\]

(b) Jacky's weight lies between 60.16 kg and 67.84 kg

and Johnson's weight lies between 59.5 kg and 60.5 kg

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4. The figure below shows a solid consisting of a cone, a cylinder and a hemisphere, all of which have the same base radius. The heights of the cone and the cylinder are 6 cm. Find the volume of the solid in terms of $\pi$. 

$$\text{Volume of the solid} = \frac{2}{3} \pi 6^2 \times 6 + \pi 6^2 \times 6 + \frac{4}{3} \pi 6^3 \times \frac{1}{2}$$

$$= 432 \pi \text{ cm}^3$$
5. If the equation \( m^2x^2 + (1 + m)x + 16 = 0 \) has repeated roots, find the value(s) of \( m \).

\[
\Delta = (1 + m)^2 - 4m^2(16) = 0
\]
\[
1 + 2m + m^2 - 64m^2 = 0
\]
\[
63m^2 - 2m - 1 = 0
\]
\[
m = \frac{1}{7} \text{ or } -\frac{1}{7}
\]

(4 marks)

\[ \text{M} \text{ for } b^2 < 4ac \]

1A

1A + 1A

6. The sum of three consecutive integers is greater than 13 and less than 21.

Find the numbers.

(5 marks)

Let the three numbers be \( x, x+1, x+2 \)

\[
13 < x + (x+1) + (x+2) < 21
\]
\[
13 < 3x + 3 < 21
\]
\[
10 < 3x < 18
\]
\[
\therefore \frac{10}{3} < x < 6
\]

The numbers are 4, 5, 6 or 5, 6, 7

1A + 1A
7. Danny deposits $10000 in a bank at an interest rate 4% p.a.

(a) If the interest is calculated on the basis of compound interest paid yearly, how much will he receive after 10 years?

(b) If the interest is calculated on the basis of simple interest, how long should he deposit the money so that the interest received is the same as found in (a).

(Correct your answer to the nearest integer.)

\[ \text{a)} \quad \text{The interest} \]
\[ = 10000 \times (1 + 0.04)^{10} - 10000 \]
\[ = 4802.428 \]
\[ = $4802, \text{ corr. to the nearest integer} \]

\[ \text{b)} \quad \text{Let } n \text{ be the number of years} \]
\[ 10000 \times 0.04 \times n = 4802.428 \]
\[ n = 12, \text{ corr. to the nearest integer} \]
8. A and B are points on the polar coordinate plane with pole O below.

(a) Write down the polar coordinates of A and B.

(b) Find the area of ΔAOB.

(c) Find the length of AB.

(a) \( A = (2, 60^\circ) \)

\( B = (3, 330^\circ) \)

(b) Area of \( ΔAOB = \frac{1}{2} \cdot OA \cdot OB \cdot \sin 90^\circ = \frac{1}{2} \cdot 2 \cdot 3 \cdot 1 = 3 \)

(c) \( AB^2 = OA^2 + OB^2 = 2^2 + 3^2 = 4 + 9 = 13 \)

\( AB = \sqrt{13} \)
9. Figure below shows the box-and-whisker diagram for the Mathematics test marks of a class.

\[ \text{Marks} \]

25 35 45 55 65 75 85 95

(a) Find the median, range and inter-quartile range of the test marks of the class.

(b) There are 43 students in the class. How many students are there who scores 65 or less in such the test?

(c) The teacher modulates the marks by adding certain fixed marks to every student so that the adjusted highest mark becomes 100. Draw the revised box-and-whisker diagram.

(a) \( \text{Median} = 65 \text{ marks} \)

\( Q_1 = 45 \text{ kg}, \ Q_3 = 75 \text{ kg} \)

Inter-quartile range = \( 75 - 45 = 30 \text{ kg} \)

Range = \( 85 - 25 = 60 \text{ kg} \)

(b) \( \text{Median} = 65 = \left( \frac{43 + 1}{2} \right) \text{ th datum} \)

\( 65 = 22 \text{ datum} \)

\( \therefore 22 \text{ students who score 65 marks or less} \)

(c) Each mark would increase 15 marks

40 50 60 70 80 90 100 110 120

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10. In the figure below, \( A(1,4), B(13,8) \) and \( C(10,-3) \) are vertices of \( \triangle ABC \). \( AM \) is a median of \( \triangle ABC \) and \( PQ \) is the perpendicular bisector of \( AB \). \( AM \) and \( PQ \) intersect at \( F \).

(a) Find the equation of \( AM \). (3 marks)

(b) Find the equation of \( PQ \). (3 marks)

(c) \( BF \) is produced to meet \( AC \) at \( G \) (not shown in the figure). Is \( BG \) an altitude of \( \triangle ABC \)? Explain your answer. (3 marks)

(a) \( M = \left( \frac{1+10}{2}, \frac{4+3}{2} \right) = \left( \frac{11}{2}, \frac{7}{2} \right) \)

equation of \( AM \) : \( \frac{y-4}{x-1} = \frac{\frac{7}{2} - 4}{\frac{11}{2} - 1} \)

\( \frac{y-4}{x-1} = -\frac{1}{3} \)

\( 3y - 12 = -x + 1 \)

\( 3x + y - 29 = 0 \)

(b) slope of \( AB \) = \( \frac{\frac{7}{2} - 4}{\frac{11}{2} - 1} = \frac{3}{5} \)

slope of \( PQ \) = \( -3 \)

mid-p of \( AB \) = \( \left( \frac{\frac{11}{2} + 13}{2}, \frac{\frac{7}{2} + 8}{2} \right) = \left( 7, \frac{15}{2} \right) \)

\( \frac{y - \frac{15}{2}}{x - \frac{17}{2}} = -3 \)

\( 3x + y - 27 = 0 \)

(c) \( \begin{cases} x + \frac{7}{2} y - 29 = 0 \\ 3x + y - 27 = 0 \end{cases} \)

\( F = \left( \frac{8}{3}, \frac{7}{3} \right) \)

slope of \( BF \) = \( -\frac{\frac{7}{3} - 4}{\frac{8}{3} - 1} = \frac{1}{3} \)

slope of \( AC \) = \( -\frac{\frac{7}{9} - 4}{\frac{8}{9} - 10} = \frac{7}{9} \)
11. The following is a sequence of figures formed by white chess and black chess.

1st figure 2nd figure 3rd figure 4th figure

\[\text{Number of white chess} \quad \text{Number of black chess}\]
\[
\begin{array}{|c|c|c|}
\hline
1^{st} \text{ figure} & 1 & 0 \\
2^{nd} \text{ figure} & 1 & 3 \\
3^{rd} \text{ figure} & 6 & 3 \\
4^{th} \text{ figure} & 6 & 10 \\
5^{th} \text{ figure} & 15 & 10 \\
6^{th} \text{ figure} & 15 & 21 \\
\hline
\end{array}
\]

(b) Find the number of white chess and the number of black chess in the 20th figure.

(i) The number of white chess

\[= 1 + 2 + 3 + \ldots + 10 \text{ terms}\]
\[= \frac{10}{2} (1 + 2 + 3 + 4 + \ldots + 10)\]
\[= 10 \times 55\]
\[= 550\]

(ii) The number of black chess

\[= 3 + 7 + \ldots + 10 \text{ terms}\]
\[= \frac{10}{2} (3 + 4 + 5 + \ldots + 10)\]
\[= 5 \times 47\]
\[= 235\]
Section B (33 marks)

Answer any THREE questions in this section and write your answers in the space provided. Each question carries 11 marks.

13. Let \( f(x) = x^3 - 7x + k \), where \( k \) is a constant. It is given that \( (x + 3) \) is a factor of \( f(x) \).

(a) Find the value of \( k \) and hence factorize \( f(x) \). (4 marks)

(b) Let \( g(x) = x^3 - 3x + 2 \).

(i) Express \( g(x) \) in the form of \( (x - h)^2 + k \) where \( h \) and \( k \) are constants.

(ii) Figure below shows the graph of \( y = g(x) \).

(1) Determine the coordinates of A, B, C and V.

(2) Solve \( g(x) \geq 0 \) graphically. (5 marks)

(c) Using the above results, solve \( f(x) \geq 0 \) for \( x \geq 0 \). (2 marks)

\[
\begin{align*}
\text{(a)} & \quad f(-3) &= -27 - 7(-3) + k = 0 \\
& \quad k &= 6 \\
\int & \quad f(x) &= x^3 - 7x + 6 = (x + 3)(x^2 - 3x + 2) \\
\int & \quad f(1) &= 1 - 7 + 6 = 0 \\
\int & \quad f(x) &= x^3 - 7x + 6 = (x-2)(x+3)(x-1) \\
\end{align*}
\]
12. An examination paper has two parts. To obtain a certificate, a candidate needs to pass at least one part. The probabilities of passing part I and part II are \( p \) and \( \frac{2}{3} p \) respectively.

(a) Find, in terms of \( p \),

(i) the probability of obtaining a certificate, and

(ii) the probability of passing part I, given that a candidate is known to obtain the certificate.

(b) Is it possible that the probability in (a)(ii) is 0.5? Explain your answer. (3 marks)

\[ \text{a)(i) } P(\text{obtaining a certificate}) = 1 - P(\text{fail in both parts}) \]
\[ = 1 - (1 - p)(1 - \frac{2}{3}p) \]
\[ = 1 - (1 - \frac{4}{3}p + \frac{2}{3} p^2) \]
\[ = \frac{5}{3}p - \frac{2}{3} p^2 \]

\[ \text{a)(ii) } P(\text{passing part I given that the candidate obtained the cert.}) \]
\[ = \frac{\frac{3}{3}p - \frac{2}{3}p^2}{\frac{5}{3}p - \frac{2}{3}p} = \frac{\frac{3}{5}}{p} = \frac{3}{5 - 2p} \]

\[ \frac{3}{5 - 2p} - \frac{2}{3}p \]

\[ 6 = 5p - 2p^2 \]
\[ 2p^2 - 5p + 6 = 0 \]
\[ \Delta = 25 - 4(2)(6) < 0 \]

\[ \therefore \text{The probability can't be 0.5} \]
(b) \( g(x) = \left( x^2 - 3x + \left( \frac{3}{2} \right)^2 \right) - \left( \frac{1}{2} \right)^2 + 2 \)

\[
= (x - \frac{3}{2})^2 - \frac{1}{4}
\]

\[ V = \left( \frac{3}{2}, \frac{1}{4} \right) \]

\( g(x) = (x-2)(x-1) \)

\( x = 1 \text{ or } 2 \)

\( A = (1,0) \quad B = (2,0) \)

When \( x = 0, \ y = ? \)

\( C = (0,2) \)

(2) \( g(x) \geq 0 \)

\( x \leq 1 \) or \( x \geq 2 \)

(c) \( f(x) = (x-1)(x-2)(x+3) \geq 0 \) for \( x \leq 0 \)

\( (x-1)(x-2) > 0 \)

By using part (b)

\( 0 < x \leq 1 \) or \( x > 2 \)
14. A food company produces fruit punch by mixing orange juice, lemon juice and pineapple juice. The volume of each bottle of fruit punch is 2 litres. The nutrients and the costs of three types of juice are shown in the table below:

<table>
<thead>
<tr>
<th></th>
<th>Orange juice</th>
<th>Lemon juice</th>
<th>Pineapple juice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vitamin C (units/litre)</td>
<td>5</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>Sugar (units/litre)</td>
<td>8</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Cost (dollars/litre)</td>
<td>1.3</td>
<td>2.2</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Suppose the amounts of orange juice, lemon juice and pineapple juice in each bottle of fruit punch are \( x \) litres, \( y \) litres and \( z \) litres respectively.

(a) Express \( z \) in terms of \( x \) and \( y \). 

(b) For the sake of health, each bottle of fruit punch must contain at least 10 units of Vitamin C and at most 11.5 units of sugar.

(i) Show that

\[
\begin{align*}
  x + 2y & \geq 2 \\
  2x - 2y & \leq 1 \\
  x + y & \leq 2 \\
  x, y & \geq 0
\end{align*}
\]

(ii) In the figure below, shade the region that satisfies the constraints in (i).

![Diagram](image_url)
(c) Determine the amounts of orange juice, lemon juice and pineapple juice in each bottle of fruit punch so that its cost is the minimum.

\[ x + y + z = 2 \]
\[ 3 = 2 - x - y \] \hspace{1cm} \text{IA}
\[ 5x + 6y + 4z = 710 \]
\[ 4x + 6y + 4(2 - x - y) = 710 \] \hspace{1cm} \text{IM}
\[ x + 2y \geq 2 \]
\[ 8x + 2y + 5z \leq 11.5 \]
\[ 8x + 2y + 5(2 - x - y) \leq 11.5 \] \hspace{1cm} \text{IM}
\[ 2x - 2y \leq 1 \]

\[ \therefore \frac{3}{2} \leq 0 \quad \text{and} \quad x + y + z \leq 2 \] \hspace{1cm} \text{IM}
\[ \therefore x + y \leq 2 \]

\[ C = 1.3x + 2.2y + 1.5z \]
\[ = 1.3x + 2.2y + 1.5(2 - x - y) \]
\[ = 0.2x + 0.7y + 3 \] \hspace{1cm} \text{IA}

At \((0, 1)\) \hspace{0.5cm} \[ C = 0.2x + 0.7 + 3 = 3.7 \] \hspace{1cm} \text{IM}
At \((0, 2)\) \hspace{0.5cm} \[ C = 4.4 \]
At \((1, 2)\) \hspace{0.5cm} \[ C = 3.15 \]
At \((1.25, 0.75)\) \hspace{0.5cm} \[ C = 3.275 \]

\[ \therefore \text{The cost of a bottle of soft drink is a minimum when} \quad x = 1, \ y = 0.5 \quad \text{and} \quad z = 0.5 \] \hspace{1cm} \text{IA}
15. In the figure below, $ABC$ is a right-angled isosceles triangle with $\angle BAC = 90^\circ$. $D$ is the mid-point of $BC$. $BFE$ is a straight line.

(a) Prove that $\triangle ABD \cong \triangle ACD$. (3 marks)

(b) Prove that $A, B, D$ and $F$ are concyclic. (6 marks)

(c) Prove that $AF \perp BE$. (2 marks)

(a) $AB = AC$ (given)

$BD = DC$ (given)

$AD = AD$ (common)

$\triangle ABD \cong \triangle ACD$ (SSS)

(b) Let $\angle ABF = \theta$

$\angle BAE = 180^\circ - 90^\circ - \theta$ ($\angle$ sum of $\triangle$)

$= 90^\circ - \theta$

$\angle EFC = 180^\circ - \angle AEF$ (adj $\angle$s of str. $\triangle$)

$= 90^\circ + \theta$

$\angle BDF = \angle EFC = 90^\circ$ (ext. $\angle$, cyclic quad)

$\therefore \angle ADB = \angle ADC$

$\therefore \angle ADB = \angle ADC$

$\angle ADB + \angle ADC = 180^\circ$ (adj $\angle$s of str. $\triangle$)

$\angle ADB = \angle ADC = 90^\circ$
\angle ADF = \angle BDF - \angle BDA  
\begin{align*}
&= 90^\circ + \theta - 90^\circ \\
&= \theta \\
\therefore \angle ADF &= \angle ADF \\
\therefore A, B, D \text{ and } F \text{ are concyclic (Converse of } Ls \text{ in the same segment) }
\end{align*}

c) \therefore ABDF are concyclic  
\begin{align*}
\therefore \angle DBA = \angle DBA = 90^\circ \quad (Ls \text{ in the same segment}) \\
\therefore AF \perp BE \\
\end{align*}

Alternative method (b) 

\angle ABC = \angle ACD = 45^\circ \\
\begin{align*}
\angle BFD &= 90^\circ \quad (\text{ext. cyclic quad}) \\
\angle BAD &= 180^\circ - 45^\circ - 90^\circ \\
&= 45^\circ \\
\therefore \angle BFD \\
\therefore \text{BAFD is a cyclic quad} \\
\end{align*}

---

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16. In figure (a), A, Q, and S are points on the horizontal ground. PQ and RS are two vertical poles where PQ = 20 m. The angles of elevation of P and R from A are 30° and 25° respectively. The compass bearings of A from Q and S are S42°E and S75°E respectively.

(a) (i) Find the lengths of AQ, AS and QS.

(ii) Using Heron’s formula, find the area of ΔAQS.

(b) H is a point on PA such that its height HK is 8m (see figure (b)). RH is produced to meet the horizontal ground at the point B. Find the compass bearing of B from S and the length of BS.

\[
\text{In } \triangle APCQ, \quad \tan 30^\circ = \frac{20}{AQ} \quad \Rightarrow AQ = \frac{20}{\tan 30^\circ} \approx 34.64 \text{ m, corr to 3 sig. fig.}
\]

\[
\angle QSA = 180^\circ - 75^\circ = 105^\circ
\]

\[
\text{In } \triangle AQS, \quad \frac{AS}{\sin 42^\circ} = \frac{34.61 \text{ m}}{\sin 105^\circ} \quad \Rightarrow AS = 23.19 \text{ m, corr to 3 sig. fig}
\]
\[ \angle A Q S = 71^\circ - 42^\circ = 29^\circ \] (ext\ of \ \angle) \]

\[ \frac{GS}{5 \times 33^\circ} = \frac{AG}{x} \]

\[ GS = 19.5 \times 33^\circ \]

\[ = 19.5 \times 0.5324 \]

\[ = 19.5 \text{ m, conv to 3 sig fig} \]

By Heron's formula:

\[ S = \frac{1}{2} (34.6102 + 23.9744 + 19.5324) \]

\[ = 39.1352 \]

\[ \text{Area} = 226.3744 \]

\[ = 226 \text{ m}^2, \text{ conv to 3 sig fig} \]

\[ \triangle A M K \sim \triangle A P Q \ (AAA) \]

\[ \frac{AK}{AP} = \frac{MK}{PQ} \]

\[ AK = \frac{8}{0.5} \times 3 + 6 \times 10^2 \text{ m} \]

\[ = 13.8564 \times 10^2 \]

\[ = 13.8564 \times 10^2 \text{ m, conv to 3 sig fig} \]

In \triangle A M K, by cosine formula:

\[ SK = \sqrt{23.9744^2 + 13.8564^2 - 2 \times 23.9744 \times 13.8564 \times \cos 29^\circ} \]

\[ = 34.6102 \]

\[ \cos \angle K S A = \frac{19.5324^2 + 34.6102^2 - 23.9744^2}{2 \times 19.5324 \times 34.6102} \]

\[ = 31.374195^\circ \]

\[ \angle Q S B = 180^\circ - 31.374195^\circ = 148.6258^\circ \text{ conv to 3 sig fig} \]

The compass bearing of B from S is N 73.6^\circ E

\[ \angle BS = x \text{ m} \]

\[ \frac{BK}{RS} = \frac{8}{2.6} \]

\[ x = 14.9754 \times \frac{8}{2.6} \]

\[ x = 50.8749 \text{ m} \text{ conv to 3 sig fig} \]

End of Paper