SECTION A  Short Questions. (80 marks)

1. Simplify \(\frac{x^{\frac{1}{2}} \sqrt{x^2}}{\sqrt[4]{x^0}}\) and give the answer with positive indices. (6 marks)

\[
\begin{align*}
\frac{x^{\frac{1}{2}} \sqrt{x^2}}{\sqrt[4]{x^0}} &= \frac{x^{\frac{1}{2}} \cdot x^2}{\sqrt[4]{1}} \\
&= x^{\frac{1}{2} + 2} \\
&= x^{\frac{5}{4}} \\
&= x^{\frac{10}{8}} \\
&= \sqrt{x^5}
\end{align*}
\]

2. If \(a(x^3 - x) + b(x^5 + x) = -4x^2 + 10,\) find \(a\) and \(b.\) (6 marks)

\[
\begin{align*}
ax^3 - ax + bx^5 + bx &= -4x^2 + 10 \\
(a + b)x^5 + (-a + b)x &= -4x^2 + 10
\end{align*}
\]

Equating the coefficients:

\[
\begin{align*}
a + b &= -4 \quad (1) \\
-a + b &= 10 \quad (2)
\end{align*}
\]

(1) + (2):

\[2b = 6\]

\[b = 3\]

(1) - (2):

\[2a = -14\]

\[a = -7\]

3. Simplify \(\frac{2 \log x}{\frac{1}{2} \log x^3 - \log x^2}\) (6 marks)

\[
\begin{align*}
\frac{2 \log x}{\frac{1}{2} \log x^3 - \log x^2} &= \frac{2 \log x}{\frac{3}{2} \log x - 2 \log x} \\
&= \frac{2 \log x}{\frac{3}{2} \log x - 2 \log x} \\
&= \frac{2 \log x}{(\frac{3}{2} - 2) \log x} \\
&= \frac{2 \log x}{-\frac{1}{2} \log x} \\
&= -4
\end{align*}
\]

4. Solve \(4^{2x + 1} = 4^x + 6\) \(\text{ (7 marks)}\)

\[
\begin{align*}
4^{2x + 1} &= 4^x + 6 \\
4 \cdot 4^x &= 4^x + 6 \\
4 \cdot 4^2 &= 4^2 + 6 \\
3 \cdot 4^2 &= 6 \\
4^2 &= 2 \\
4^x &= 2 \\
4x &= 1 \\
\frac{4x}{4} &= \frac{1}{4} \\
x &= \frac{1}{4}
\end{align*}
\]
5. Solve the quadratic equation $3x^2 + 2x - 3 = 0$. Leave the answers in surd form.

(6 marks)

\[
x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-3)}}{2 \times 3}
\]

\[
x = \frac{-2 \pm \sqrt{4 + 36}}{6}
\]

\[
x = \frac{-2 \pm 2\sqrt{10}}{6}
\]

\[
x = -1 \pm \frac{\sqrt{10}}{3}
\]

6. Solve \(\begin{align*} y &= x^2 + x - 2 \\ y &= 3x + 1 \end{align*}\) (9 marks)

\(y = x^2 + x - 2 \quad \text{(1)}\)

\(y = 3x + 1 \quad \text{(2)}\)

Sub. (1) into (2) \(x^2 + x - 2 = 3x + 1\)

\(x^2 - 2x - 3 = 0\)

\((x - 3)(x + 1) = 0\)

\(x = 3 \quad \text{or} \quad x = -1\)

Sub. \(x = 3 \) into (2) \(y = 3(3) + 1\)

\(= 10\)

Sub. \(x = -1 \) into (2) \(y = 3(-1) + 1\)

\(= -2\)

\(\therefore (x, y) = (3, 10), (-1, -2)\)

7. (a) Solve $5^n = 15$. Correct the answer to 4 decimal places.
(b) Solve \(\log(x - 2) + \log(2x - 3) = 1\). (10 marks)

(a) \(5^n = 15\)

\(\log 5^n = \log 15\)

\(n \log 5 = \log 15\)

\(n = \frac{\log 15}{\log 5}\)

\(n = 1.6826 \approx 1.6826\)

(b) \(\log(x - 2) + \log(2x - 3) = 1\)

\(\log(x - 2)(2x - 3) = 1\)

\((x - 2)(2x - 3) = 10\)

\(2x^2 - 7x + 6 - 10 = 0\)

\(2x^2 - 7x - 4 = 0\)

\((2x + 1)(x - 4) = 0\)

\(x = -\frac{1}{2}, \quad \text{or} \quad x = 4\)

\(\text{(rejected)}\)
8. Consider the equation \( x^2 + 4kx + 16k + 20 = 0 \) \( \ldots \ldots \) (*)
(a) Express the discriminant of equation (*) in terms of \( k \).
(b) Hence, find the values of \( k \) if the equation (*) has two equal roots. 

\[ \begin{align*}
(\text{a}) & \quad \text{The discriminant} \\
& \quad = (4k)^2 - 4(16k + 20) \\
& \quad = 16k^2 - 64k - 80 \\
& \quad \text{If (*) has two equal roots,} \\
\text{Discriminant of (*)} &= 0 \\
16k^2 - 64k - 80 &= 0 \\
4k^2 - 16k - 20 &= 0 \\
(k - 5)(k + 4) &= 0 \\
k &= 5 \quad \text{or} \quad k = -4
\end{align*} \]

9. Figure 1 shows the graph of \( y = x^2 - 3x + k \), where \( k \) is a constant.
(a) Find the value of \( k \).
(b) Using the graph, solve the following inequalities.
(i) \( x^2 - 3x + 2 \geq 2 \)
(ii) \( x^2 - 3x \leq 4 \)

\[ (10 \text{ marks}) \]

\[ \begin{align*}
(\text{a}) & \quad \text{Sub. (0,4) into the equation} \\
& \quad 4 = c^2 - 3c + k \\
& \quad k = 4 \\
(\text{b}) & \quad \text{The line } y = 2 \text{ should be added} \\
& \quad (i) \quad \text{From the graph, } x \leq 1 \text{ or } x \geq 2 \\
& \quad (ii) \quad x^2 - 3x \leq 4 \quad \text{can be written as} \\
& \quad x^2 - 3x + 4 \leq 0 \\
& \quad \text{In order to solve this equation,} \\
& \quad \text{the line } y = 0 \text{ should be added} \\
& \quad \text{From the graph, } -1 \leq x \leq 4
\end{align*} \]
10. Given \( f(x) = 2x^2 - ax - 2 \).
(a) When \( f(4) = 10 \) find the value of \( a \).
(b) When \( f(x) \) is divided by \( (x + q) \), the remainder is 2q. Find the value(s) of \( q \).

(l marks)

\[
\begin{align*}
\text{(a)} & \quad f(4) = 10 \\
& \quad 2(4)^2 - a(4) - 2 = 10 \\
& \quad 32 - 4a - 2 = 10 \\
& \quad 4a = 20 \\
& \quad a = 5
\end{align*}
\]

\[
\begin{align*}
\text{(b)} & \quad f(x) = 2x^2 - 5x - 2 \\
& \quad f(-\frac{3}{2}) = 2\left(-\frac{3}{2}\right)^2 - 5\left(-\frac{3}{2}\right) - 2 = 2\left(-\frac{3}{2}\right)^2 + 5\left(-\frac{3}{2}\right) - 2 = 2\cdot\frac{9}{4} - \frac{15}{2} - 2 = \frac{9}{2} + \frac{15}{2} - 2 = 0 \quad \left(2\cdot\frac{9}{2} - 1\right)
\end{align*}
\]

\[
\begin{align*}
q &= \frac{1}{2} \\
& \text{or} \quad q = -2
\end{align*}
\]

SECTION B Long Questions. (40 marks)

11. Figure 2 shows the graph of \( y = 2x^2 - 8x + c \). The y-intercept of the graph is 15.

\[
\begin{align*}
\text{(a)} & \quad \text{Find the value of } c. \\
\text{(b)} & \quad \text{Express the equation of the graph in the form } y = a(x - h)^2 + k. \\
\text{(c)} & \quad \text{Find the vertex and axis of symmetry of the graph.} \\
\text{(d)} & \quad \text{If the graph of } y = 2x^2 - 8x + c \text{ is translated 5 units to the right and 10 units downwards, using the result of (b), write down the equation of the image.} \\
\text{(e)} & \quad \text{By how many units should the graph be } y = 2x^2 - 8x + c \text{ translated downwards, so that the new graph touches the x-axis at one point, point } P, \text{ only? Write down the equation of the new graph.} \\
\text{(f)} & \quad \text{Find the coordinates of } P \text{ and the new y-intercept.}
\end{align*}
\]

\[
\begin{align*}
\text{(a)} & \quad \text{y-intercept} = c \\
& \quad c = 15
\end{align*}
\]

\[
\begin{align*}
\text{(b)} & \quad y = 2x^2 - 8x + 15 \\
& \quad = 2(x^2 - 4x + 15) \\
& \quad = 2(x^2 - 4x + 3^2 - 3^2 + 15) \\
& \quad = 2(x^2 - 4x + 3^2) + 15 \\
& \quad = 2[(x-2)^2 - 2^2 + 15] \\
& \quad = 2(x-2)^2 - 4 + 15 \\
& \quad = 2(x-2)^2 + 11
\end{align*}
\]

\[
\begin{align*}
\text{(c)} & \quad \text{Vertex } (2, 11) \\
\text{Axis of symmetry is } x = 2
\end{align*}
\]
12. (a) Given two polynomials $x^3 - 7x^2 + 4x + 12$ and $x^3 + 3x^2 - 6x - 8$

(i) Show that $x - 2$ is a common factor of the two polynomials.

(ii) Hence factorize the two polynomials completely.

(iii) Hence solve the equation \[
\frac{1}{x^3 - 7x^2 + 4x + 12} + \frac{1}{x^3 + 3x^2 - 6x - 8} = 0
\]

(b) Given $h(x)$ is a polynomial in $x$. When $h(x)$ is divided by $x + 3$, the remainder is $-13$. When $h(x)$ is divided by $x - 3$, the remainder is $11$.

Find the remainder when $h(x)$ is divided by $x^2 - 9$.

(b) (i) \[f(x) = x^3 - 7x^2 + 4x + 12\]

\[g(x) = x^3 + 3x^2 - 6x - 8\]

\[f(2) = 2^3 - 7(2)^2 + 4(2) + 12 = 0\]

\[g(2) = 2^3 + 3(2)^2 - 6(2) - 8 = 0\]

(iii) $x^3 - 7x^2 + 4x + 12$

$= (x - 2)(x^2 - 5x - 6)$

$= (x - 2)(x + 1)(x - 6)$

\[x^3 + 3x^2 - 6x - 8\]

$= (x + 3)(x^2 + 6x + 4)$

$= (x - 2)(x + 1)(x + 4)$

(iii) \[
\frac{1}{x^3 + 3x^2 - 6x - 8} + \frac{1}{x^3 - 7x^2 + 4x + 12} = 0
\]

\[
\frac{1}{(x - 2)(x + 1)(x - 6)} + \frac{1}{(x - 2)(x + 1)(x + 4)} = 0
\]

\[
\frac{1}{(x - 2)(x + 1)(x - 6)(x + 4)} = 0
\]

\[
\frac{1}{(x - 2)(x + 1)(x - 6)(x + 4)} = 0
\]
12 (a) \( x - 2 = 0 \)
\[ x = 2 \]

12 (b)

Let the remainder be \( a \times x + b \) and the quotient be \( Q(x) \):

\[ R(x) = (x^2 - 9)Q(x) + a \times x + b \]
\[ R(-3) = -12 \]
\[ (63)^2 - 91[Q(-3)] - 3a + b = -12 \]
\[ -3a + b = -12 \quad -(1) \]

\[ R(3) = 11 \]
\[ (3)^2 - 91[Q(3)] + 3a + b = 11 \]
\[ 3a + b = 11 \quad -(2) \]

\[ (1) + (2) \quad 2b = -2 \]
\[ b = -1 \]

\[ (2) - (1) \quad 6a = 24 \]
\[ a = 4 \]

\[ \therefore \text{The remainder is } 4x - 1. \]