Queen's College
Yearly Examination 2007 – 2008

ADDITIONAL MATHEMATICS
Question Answer Book

Secondary 4
Date: 20th June, 2008
Time: 10:30 am – 12:30 pm (2 hours)

This paper must be answered in English

1. Write your class and class number in the space provided on this cover.

2. This paper consists of TWO sections, Section A and Section B. Section A carries 52 marks and Section B carries 48 marks.

3. Answer ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Graph paper and supplementary answer sheets will be supplied on request.

4. All working must be clearly shown.

5. Unless otherwise specified, numerical answers must be exact.

6. In this paper, vectors may be represented by bold-type letters such as \( \mathbf{u} \), but candidates are expected to use appropriate symbols such as \( \overrightarrow{u} \) in their working.

7. The diagrams in the paper are not necessarily drawn to scale.
1. Let \( T_n = \frac{n(n+2)}{(n+1)^2} \), where \( n \) is a positive integer.

Prove by mathematical induction that

\[ T_1 \times T_2 \times \ldots \times T_n = \frac{n+2}{2(n+1)} \]

for all \( n \). (6 marks)

2. Let \( x \) be a real number.

(a) If \( y = x^2 - 6x - 3 \), find the range of possible values of \( y \).

(b) If \( z = \frac{60}{|x^2 - 6x - 3| + 1} \), find the range of possible values of \( z \). (6 marks)
3. In Figure 1, OA, OB and OC are three mutually perpendicular edges of a tetrahedron OABC. D is a point on AB such that CD is perpendicular to AB. OA = OB = 4 cm and OC = 3 cm. Find, correct to 3 significant figures.
   (a) the length of CD,
   (b) the angle between the planes ABC and OAB.

4. Find the general solution of \( \sin^2 x = 5 \cdot 2 \cos x \) correct to the nearest 0.1°.
5. Given that \(7 \cos x - \sin x = R \cos(x + \alpha)\), where \(R > 0\) and \(\alpha\) is acute, find the values of \(R^2\) and \(\tan \alpha\). Hence

(a) solve the equation \(7 \cos x - \sin x = 5\) correct to the 3 significant figures, giving all solutions in the interval \(-\pi < x < \pi\),

(b) write down the least value of \(\frac{1}{(7 \cos x - \sin x)^2}\). (7 marks)

6. In the binomial expansion of \(\left(1 + \frac{x}{n}\right)^n\) in ascending powers of \(x\), the coefficient of \(x^3\) is \(\frac{7}{16}\). Given that \(n\) is a positive integer,

(a) find the value of \(n\),

(b) evaluate the coefficient of \(x^3\) in the expansion. (6 marks)
7. Let \( \mathbf{a} \), \( \mathbf{b} \) be two vectors such that \( |\mathbf{a}| = 4 \), \( |\mathbf{b}| = 5 \) and \( \mathbf{a} \cdot \mathbf{b} = 10 \).

(a) Find the angle between \( \mathbf{a} \) and \( \mathbf{b} \).

(b) If the vectors \( \mathbf{a} + 2 \mathbf{b} \) and \( \mathbf{a} + k \mathbf{b} \) are perpendicular to each other, find the value of \( k \).

(6 marks)

8. In Figure 2, the tangent at \( C \) cuts \( AB \) produced at \( T \). Given \( AB = 10 \), find \( CT \) correct to 3 significant figures.

(7 marks)
In Figure 3, \( \triangle ABC \) is isosceles with \( AB = AC = 1 \). \( D \) is a point on \( AC \) such that \( AD = DB = BC \).

(a) Show that \( \angle ACB = 72^\circ \).

Hence show that \( BC = 2 \cos 72^\circ \). \hspace{1cm} (5 marks)

(b) Show that \( \triangle ABC \sim \triangle BCD \).

Hence deduce that \( 4 \cos^2 72^\circ = 1 - 2 \cos 72^\circ \). \hspace{1cm} (6 marks)

(c) By expressing \( \cos 2\theta \) in terms of \( \cos \theta \) and without solving \( \cos 72^\circ \) in (b), evaluate \( \cos 72^\circ + \cos 144^\circ \). \hspace{1cm} (5 marks)
In this question, numerical answers should be correct to three significant figures.

**Figure 4**, shows a net which can be folded into a pyramid $VABCD$ with rectangular base $ABCD$ and with $V$ vertically above $A$ by gluing $V_1, V_2, V_3$ and $V_4$ into a single point $V$. Suppose $AB = 6$, $AD = 8$ and $\angle V_1DA = 35^\circ$.

(a) Other than the angles of $ABCD$, name 5 other right angles in the figure. (3 marks)

(b) Find $\angle V_2BA$. (5 marks)

(c) Suppose the net is folded into pyramid $VABCD$.

(i) Name all planes of the pyramid which are perpendicular to each other.

(ii) Find the volume of pyramid.

(iii) Find the angle between $VC$ and the plane $ABCD$. (8 marks)
In Figure 5, \(A, B, C\) and \(D\) are four points on a plane such that \(\overrightarrow{OA} = 6\mathbf{i} + 9\mathbf{j}\).

\(\overrightarrow{OB} = \lambda \mathbf{i} + 2\mathbf{j}\), \(\overrightarrow{OC} = 5\mathbf{i} + \mathbf{j}\) and \(\overrightarrow{OD} = 2\mathbf{i} + 3\mathbf{j}\).

(a) If \(O, C\) and \(B\) are collinear, find the value of \(\lambda\). (3 marks)

(b) \(AB\) and \(DC\) are produced to meet at \(P\).

Suppose \(AB : BP = 1 : m, DC : CP = 1 : n\).

(i) Express \(\overrightarrow{OP}\) in terms of \(m, \mathbf{i}\) and \(\mathbf{j}\). (8 marks)

(ii) Express \(\overrightarrow{OP}\) in terms of \(n, \mathbf{i}\) and \(\mathbf{j}\).

(iii) Hence evaluate \(m\) and \(n\). (8 marks)

(c) Using the results obtained in (b), find \(\overrightarrow{OP}\) and \(\overrightarrow{DP}\). Deduce that \(\overrightarrow{OA} \perp \overrightarrow{DP}\). (5 marks)