**QUEEN'S COLLEGE**
**Half-yearly Examination, 2008-2009**

**S.1 MATHEMATICS PAPER 1**

**MARKING SCHEME**

**General Marking Instructions**

1. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question.

2. In the marking scheme, marks are classified into the following three categories:
   - ‘M’ marks awarded for correct methods being used;
   - ‘A’ marks awarded for the accuracy of the answers;
   - Marks without ‘M’ or ‘A’ awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. Marks may be deducted for wrong units (u) or poor presentation (pp).
   a. The symbol \(u - 1\) should be used to denote 1 mark deducted for u. At most deduct 1 mark for u for the whole paper.
   b. The symbol \(pp - 1\) should be used to denote 1 mark deducted for pp. At most deduct 3 marks for pp for the whole paper. For similar pp, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same pp.
   c. At most deduct 1 mark in each question. Deduct the mark for u first if both marks for u and pp may be deducted in the same question.
   d. In any case, do not deduct any marks for pp or u in those steps where candidates could not score any marks.

4. All fractional answers must be simplified.
SECTION A  Short questions.  (80 marks)

Answer ALL questions in this section and write your answers in the spaces provided.

1.  (a) Insert the correct symbol ‘<’ or ‘>’ between the two given numbers.

   (i) \(-\frac{1}{7}, -\frac{1}{4}\) \hspace{1cm} (1 mark)

   (ii) \(-\frac{3}{4}, -\frac{5}{8}\) \hspace{1cm} (1 mark)

   (iii) \(-2^2, -2^3\) \hspace{1cm} (1 mark)

(b) Arrange \(-\frac{1}{7}, -\frac{1}{4}, -\frac{3}{4}, -\frac{5}{8}, -2^2, -2^3\) in descending order. \hspace{1cm} (2 marks)

(a) (i) \(-\frac{1}{7} > -\frac{1}{4}\) \hspace{1cm} 1A

   (ii) \(-\frac{3}{4} < -\frac{5}{8}\) \hspace{1cm} 1A

   (iii) \(-2^2 > -2^3\) \hspace{1cm} 1A

(b) \(-\frac{1}{7}, -\frac{1}{4}, -2^2, -\frac{3}{4}, -\frac{5}{8}, -2^3\) \hspace{1cm} 2A for all correct

   -1A for ascending order

   -1A for not using given no(s).

2.  Evaluate \(5 - 2[13 - 10 \div (-2)] \div 3 - 1\) \hspace{1cm} (5 marks)

\[
5 - 2[13 - 10 \div (-2)] \div 3 - 1 \\
= 5 - 2[13 + 5] \div 3 - 1 \hspace{1cm} 1A \text{ for } [13 + (-5)] \div ... \\
= 5 - 2[18] \div 3 - 1 \\
= 5 - 36 \div 3 - 1 \text{ or } 5 - \frac{2[18]}{3} - 1 \hspace{1cm} 1A \text{ for } -36 \div 3 - ... \text{ or } -\frac{2[18]}{3} - ... \\
= 5 - 2[6] - 1 \\
= 5 - 12 - 1 \hspace{1cm} 1A \text{ for } 5 - 12 - 1 \\
= 5 - 13 \\
= -8 \hspace{1cm} 1A
\]
3. Evaluate \(-2 - \left[ -\frac{1}{3} - \frac{2}{3} \div (-6) \right] \div 1\frac{1}{2} - 1\)

\[
-2 - \left[ -\frac{1}{3} - \frac{2}{3} \times \frac{1}{6} \right] \div 1\frac{1}{2} - 1
\]

\[
= -2 - \left[ -\frac{1}{3} - \frac{1}{9} \right] \times \frac{2}{3} - 1
\]

1M for \(-\left[ -\frac{2}{9} \right] \times \frac{2}{3} \)

\[
= -2 - \left[ -\frac{3}{9} + \frac{1}{9} \right] \times \frac{2}{3} - 1
\]

1A for \(-\left[ -\frac{2}{9} \right] \times \frac{2}{3} \)

\[
= -2 - \left[ -\frac{4}{27} \right] - 1
\]

1M for \(-\left[ -\frac{4}{27} \right] \)

\[
= -3 + \frac{4}{27}
\]

1A for \(-3 + \ldots\)

\[
= -2 \frac{23}{27}
\]

1A
4. Solve \( \frac{x+4}{4} = \frac{3}{2} - \frac{2x+7}{3} \)

\[
\begin{align*}
\frac{x+4}{4} &= \frac{3}{2} - \frac{2x+7}{3} \\
12 \times \left( \frac{x+4}{4} \right) &= 12 \times \left( \frac{3}{2} - \frac{2x+7}{3} \right) \quad 1\text{A} \\
3 \times (x+4) &= 12 \times \frac{3}{2} - 12 \times \frac{2x+7}{3} \quad 1\text{M} \\
3x+12 &= 6x - 4(2x+7) \quad 1\text{A} \\
3x+12 &= 18 - 8x - 28 \quad 1\text{A} \\
3x+8x &= -10 - 12 \quad 1\text{A} \\
11x &= -22 \quad 1\text{A} \\
x &= -2 \quad 1\text{A}
\end{align*}
\]

(10 marks)
5. Mr Li has borrowed some money from a bank and will pay simple interest.

(a) If he has borrowed $6 500 and should pay back an amount of $10 400 after 4 years, find the interest rate per annum. (5 marks)

(b) If the interest rate is 12% p.a. and he should pay back an amount of $27 200 after 3 years, find the sum of money he has borrowed. (5 marks)

(a) Let the interest rate be \( r \)\% p.a.  

\[
10400 - 6500 = 6500 \times \frac{r}{100} \times 4
\]

\[
\frac{3900 \times 100}{6500 \times 4} = r
\]

\[
r = 15
\]

the interest rate is 15% p.a.

(b) Let the sum of money he has borrowed be \( P \).

\[
P \times \left(1 + \frac{12}{100} \times 3\right) = 27 200
\]

\[
P \times (1.36) = 27 200
\]

\[
P = 20 000
\]

He has borrowed $20 000.
6. Consider the sequence: \[ \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \] ............

(a) Write down the next 2 terms of the sequence. (2 marks)

(b) (i) Rewrite the first 4 terms of the above sequence into similar numerical expressions and then use an algebraic expression to represent the general term \( a_n \) of the sequence. (3 marks)

(ii) Use the result of (b)(i) to find \( a_{30} \), the 30\(^\text{th} \) term of the sequence. (2 marks)

(iii) If the \( n \)th term of this sequence is 0.02, find the value of \( n \). (3 marks)

(10 marks)

<table>
<thead>
<tr>
<th>(a) the next 2 terms are</th>
<th>1, 2, 3, 4, 5</th>
<th>2A</th>
</tr>
</thead>
</table>

(b) (i) \( a_1 / 1\text{st term} = \frac{1}{2} = \frac{1}{1+1} \)

\( a_2 / 2\text{nd term} = \frac{1}{3} = \frac{1}{2+1} \) 1M For writing \( \frac{1}{2} = \frac{1}{1+1} \) & \( \frac{1}{3} = \frac{1}{2+1} \)

\( a_3 / 3\text{rd term} = \frac{1}{4} = \frac{1}{3+1} \)

\( a_4 / 4\text{th term} = \frac{1}{5} = \frac{1}{4+1} \) 1M For writing \( \frac{1}{4} = \frac{1}{3+1} \) & \( \frac{1}{5} = \frac{1}{4+1} \)

\( \therefore \text{ the general term } a_n = \frac{1}{n+1} \) 1A

(ii) \( a_{30} \), the 30\(^\text{th} \) term = \( \frac{1}{30+1} \) 1M For substitution.

= \( \frac{1}{31} \) 1A

(iii) \( \frac{1}{n+1} = 0.02 \) 1A

\( \frac{1}{0.02} = n+1 \)

\( 50 - 1 = n \) 1M For transposing term (+1 to -1) correctly.

\( n = 49 \) 1A
7. Mr. Chan has x diamond rings and Mrs. Chan has 2x diamond rings.

(a) Write an algebraic expression to represent the total number of their diamond rings. (1 marks)

(b) If x = 12, find the total number of their diamond rings. (2 marks)

(c) Mr. Chan said that their total number of diamond rings is less than 12. Write an inequality to express this fact. (1 mark)

(d) Write down all possible values of x which satisfy what he said. (1 mark)

(e) Mrs. Chan said that their total number of diamond rings is not less than 8. Write an inequality to express what she said. (1 mark)

(f) Does x = 4 satisfy the inequality in (e)? (1 mark)

(g) Does x = 3 satisfy the inequality in (e)? (1 mark)

(h) Does x = 2 satisfy the inequality in (e)? (1 mark)

(i) Write down all possible values of x which satisfy what they both said. (1 mark)

(a) \[ x + 2x \text{ or } 3x \text{ or } x + 2x = 3x \]

(b) the total number of their diamond rings = 3x

\[ = 3(12) \]

\[ = 36 \]

(c) \[ 3x < 12 \text{ or } x + 2x < 12 \]

(d) \[ x = 0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \]

(e) \[ 3x \geq 8 \text{ or } x + 2x \geq 8 \]

(f) Yes. (1A)

(g) Yes (1A)

(h) No (1A)

(i) x = 3 (1A)
8. The diameter of a ball had been measured to be 15 cm. It was known that 4 layers of balls could be stacked up in the box and each layer contained $6 \times 2$ balls.

(a) Find the smallest possible length, width and height of the box. (6 marks)

(b) Hence, find the volume of the box and correct it to the nearest 10 000 cm$^3$. (4 marks)

(a) smallest possible length/width = $(6 \times 15)$

$= 90$ cm

(b) smallest possible volume = $90 \times 30 \times 60$

$= 162000$ cm$^3$

$\approx 160000$ cm$^3$ 

-1 pp For replacing ‘$\approx$’ with ‘=’.
9. (a) Follow each of the instructions below to find an estimated value of the expression: $337.9 + 184.4 + 266.6 + 209.8$

(i) Round off each number correct to the nearest ten. (2 marks)

(ii) Round down each number correct to the nearest ten. (2 marks)

(iii) Round up each number correct to the nearest ten. (2 marks)

(b) Which of the above methods will always give an estimation bigger than the exact value? (2 marks)

(c) A supermarket is offering a promotion such that if a customer spends $1000 or more, he/she will receive a free gift. Ann wants to buy 4 bottles of wine of prices $337.9, $184.4, $266.6 and $209.8 respectively. If she wants to have the free gift, which method in (a) should she use to estimate whether she will spend $1000 or more, why? (2 marks)

(10 marks)

(a) (i) $337.9 + 184.4 + 266.6 + 209.8$

$\approx 340 + 180 + 270 + 210$

$= 1000$

(ii) $337.9 + 184.4 + 266.6 + 209.8$

$\approx 330 + 180 + 260 + 200$

$= 970$

(iii) $337.9 + 184.4 + 266.6 + 209.8$

$\approx 340 + 190 + 270 + 210$

$= 1010$

(b) Method (iii) or Rounding up. 2A

(c) She should use method (ii) or rounding down to estimate, 1A because the other 2 methods may give an estimation bigger than the exact value. 1M
10. The cost price of a toy in a shop is $400. In May, it was marked at a price so that the profit per cent was 25%. In July, the marked price was then increased so that if the toy was sold at a 40% discount, the loss per cent was 10%.

(a) Find the marked price of the toy in May. (4 marks)

(b) Find the selling price of the toy in July. (4 marks)

(c) Find the marked price of the toy in July. (6 marks)

(d) Find the percentage increase in the marked prices from May to July. (6 marks)
11. A tram leaves the terminal with 50 passengers on it.

(a) When the tram comes to the first stop, \( \frac{3}{5} \) of the passengers on board get off the tram and 4 passengers get on it. Find the number of passengers on the tram when it leaves the first stop. (5 marks)

(b) At the second stop, \( \frac{1}{4} \) of the passengers on board get off the tram and \( y \) passengers get on it. Find (in terms of \( y \)) the number of passengers on the tram when it leaves the second stop. (5 marks)

(c) At the third stop, \( \frac{1}{3} \) of the passengers on board get off the tram and 5 passengers get on it. If the number of passengers on the tram now is only half that when the bus left the bus terminal, find the value of \( y \). (10 marks)

(20 marks)

(a) number of passengers on the bus when it leaves the first stop

\[
= \left(1 - \frac{2}{5}\right) \times 50 + 4
= \frac{3}{5} \times 50 + 4
= 24
\]

(b) number of passengers on the bus when it leaves the second stop

\[
= \left(1 - \frac{1}{4}\right) \times 24 + y
= \frac{3}{4} \times 24 + y
= 18 + y
\]

(c) \[
\left(1 - \frac{1}{3}\right)(18 + y) + 5 = \frac{1}{2} \times 50
\]

\[
\frac{2}{3} (18 + y) + 5 = 25
\]

\[
\frac{2}{3} (18 + y) = 25 - 5
\]

\[
18 + y = \frac{3}{2} \times 20
\]

\[
y = 30 - 18
\]

\[
y = 12
\]