“Big 2” is the one of the most popular local card game in Hong Kong. The game is played by 4 players. Each player is given 13 cards and take turns to give cards and the first party who finish giving his last card will be the winner.

Players can give a single, a pair, a triple, or at most five cards in every round.

The combination of the five cards given out must follow one of the following patterns, which are namely, “straight”, “Flush”, “Full house” and “Royal Straight”.

<table>
<thead>
<tr>
<th>Full House</th>
<th>![Heart 5, 5, 7, 7, 7]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight</td>
<td>![Spade 3, 4, 5, 6, 7]</td>
</tr>
<tr>
<td>Flush</td>
<td>![Diamond 2, 3, 5, 9, 10]</td>
</tr>
<tr>
<td>Straight Flush</td>
<td>![Club 6, 7, 8, 9, 10]</td>
</tr>
</tbody>
</table>

*Straight Flush” is equivalent to “Royal Flush”
One day I suddenly took the initiative to calculate the probability of getting a “Straight” in a game of Big2. I found the question quite difficult for me.

That’s why I only consider the probability of getting a pattern in a game of Show-hand.

It’s much easier to obtain the probability of getting a Flush in a game of “show hand”. The answer is given by “4 * 13C5 / 52C5 “which is equal to “1.980792317*10^-3”.

(Assuming the cards are obtained randomly from 52 cards)
Getting a "Straight".
The total number of favorable outcome is:

The probability is 4*4*4*4*4 – 4 (Straight Flush)
As there are ten patterns of Straights
i.e. {A2345 / 23456 / ... / 10-IQKA} □ 10 patterns

The probability of getting a Straight is 1020*10 / 52C5
=3.924646782*10^-3

The probability of getting a “Straight Flush” is:
40 / 52C5 = 1.539077169*10^-5. It’s really rare!

The probability of getting a “Full House” is :
13*4C3*12*4C2 / 52C5 = 1.44057623*10^-3

Analysis:
13*4C3 = number of combinations of getting a triple
12*4C2 = number of combinations of getting a pair
From my calculations, we obtain the probabilities of getting different patterns. We can see that

\[ P(\text{Straight}) > P(\text{Flush}) > P(\text{Full House}) > P(\text{Straight Flush}) \]

This may explain why
The “power” of different patterns are in such an order:

Straight Flush > Full House > Flush > Straight !!!!
I have found another interesting thing about Poker.
What happen when we randomly take 13 cards from 52 cards?
For example,
The probability of getting a Queen in a set of 13 cards:
\[ \frac{4 \times 13}{52} = 1 \]

This implies every set is \{A-1-2-3\ldots Q-K\}

Surely, it’s impractical.

All of above are done by me, hope I did not get anything wrong!